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# CONSTRUCTION OF EQUATIONS OF MOTION OF MULTIBODY SYSTEMS AND COMPUTER MODELING

***Urgency of the research.*** *Computer models mean new quality in the knowledge process. Using a computer model, the properties of the subject under investigation can be tested under different operating conditions. By experimenting with a computer model, we learn about the modelled object. We can test different machine variants without having to produce and edit prototypes.*

***Target setting.*** *The development of computer technology has expanded the possibility of solving mathematical models and allowed to gradually automate the calculation of mathematical model equations. It is necessary to insert appropriate inputs of the mathematical model and monitor and evaluate the output results through the computer output device The target was to describe the mathematical apparatus required for mathematical modeling and subsequently to compile a model for computer modeling.*

***Actual scientific researches and issues analysis.*** *When formulating a mathematical model for a computer, the laws and the theory we use are always valid under more or less idealized conditions, and operate with fictitious concepts such as, material point, ideal gas, intangible spring, and the like. However, with these simplifications, we describe a realistic phenomenon where the initial assumptions are only met to a certain extent. In order for the results not to be different from the modeled reality, it is to be assumed that a good computer model arises gradually, by verifying and modifying it, which is one of the advantages of MSC Adams.*

***Uninvestigated parts of general matters defining.*** *The question of building a real manipulator model. Based on the above simulation, it is possible to build a real model.*

***The research objective.*** *Using MSC Adams to simulate multiple body systems and verify its suitability for simulating manipulator and robot models. In various versions of the assembled model we can monitor its behavior under different operating conditions.*

***The statement of basic materials.*** *In computer simulation, MSC Adams-View is used to simulate mechanical systems. It has an interactive environment for automated dynamic analysis of parameterized mechanical systems with an arbitrary structure of rigid and flexible bodies with geometric or force joints, in which act gravity, inertia, experimentally designed contact, friction, aerodynamic, hydrodynamic or electromechanical forces and have integrated control, hydraulic, pneumatic or electromechanical circuits.*

***Conclusions.*** *Working with a mathematical model on a computer opens space for specific synthesis of empirical and analytical method of scientific knowledge. Working with the computer model carries the characteristic features of classical experimentation. It represents a qualitatively new way of solving tasks that can not be experimented with on a real object. The result is the equivalence of the computer model and the object being investigated with the features and expressions chosen as essential, with accuracy sufficient to the exact purpose.*

***Keywords:*** *multi body systems; kinematic pairs; transformation matrices; software simulation.*

*Fig.: 9. References: 17.*

**Introduction.** There is a variety of machines and devices consisting of spatial mechanisms and space-bound mechanical systems. We find them in material handling machines as well as in precise mechanics products. They are also parts of various manipulators and robots. In order for the spatial mechanisms to be introduced more rapidly into practice and in order to increase their performance we need to replace the old computational practices that were largely intuitive and suitable for individual cases or small groups of mechanisms with more general, accurate and easily algorithmizable methods which are suitable for use with computing. The following chapters of the paper illustrate the matrix methods of compiling equations of motion. The matrix notation is an optimal approach due to its compactness, efficiency of applying various transformations and the suitability for useing computers in numerical calculations.

Structured mechanics were dealt with in works [1-9], more recently in [10-17]. Methods of mechanism description are described in works of Denavit and Hartenberg [5] and others. General methods of dynamic analysis of planar mechanisms are attributed to authors as Brát [1], Kozlov, Makaričev, Timofejev, Jurevič [2], Bejczy [3], Koplik, Leu [8], Haug [7], Stejskal [9], Valášek [9; 12], Shabana [10; 11] and others. In our work, after the introductory theoretical part we show simple manipulator models compiled in MSC Adams-View.

**Geometrics.** Matrix notation of the kinematics of the spatial motion of the point and the body is basis for the development of kinematic and dynamic methods of analysis and is described in the work of the authors [1-3]. Its compactness proved successful in investigating the simultaneous movements of body systems. Next, the use of theory of simultaneous movements of simple models of open kinematic chains is described. The geometry of the system of bodies in simultaneous movements is described on a manipulator model with 3 degrees of freedom of movement (3 DOF) (Fig. 1).

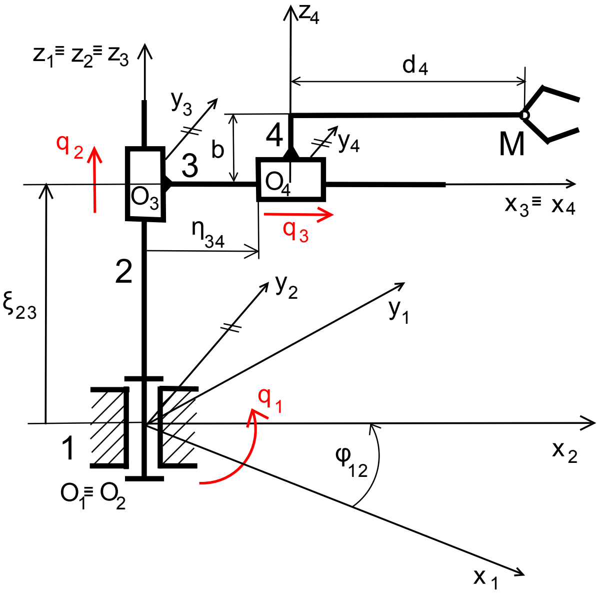


*Fig. 1. Manipulator with 3 DOF $q_1=\varphi_{12}$, $q_2=\xi_{23}$, $q_3=\eta_{34}$*

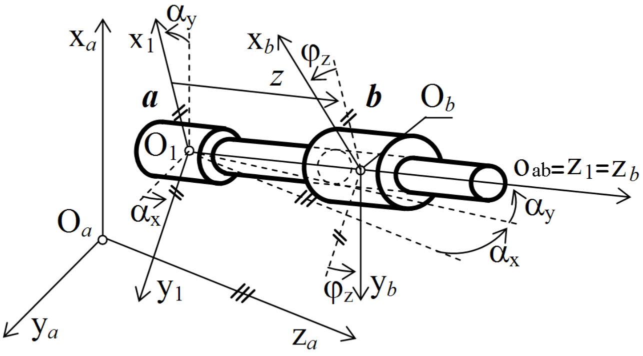


*Fig. 2. Coordinate systems*

**Model of a robot with 6 degrees of freedom of movement.** The theory of simple open kinematic chains has direct use in the kinematic analysis of the various manipulators and robots that are often referred to as such chains. The kinematic scheme of one of these robots is shown in Figure 3. The chain has seven members and has 6 degrees of freedom. It can occupy any position in the space and can therefore appropriately manipulate with any object. The movement of the member 7 is expressed by the basic decomposition to the reference point M, which is the end point. The transformation matrix is determined:

$$T_{12} = T_{rot6}\left( {}_{12k}\right)T_{rot6}\left( {}_{12p}\right). \tag{1}$$

$$T_{23} = T_{trans3}\left(\zeta_{23k}\right)T_{trans3}\left(\zeta_{23p}\right). \tag{2}$$

$$T_{34} = T_{trans3}\left(d_3\right)T_{trans2}\left(\eta_{34k}\right)T_{trans2}\left(\eta_{34p}\right). \tag{3}$$

$$T_{45} = T_{z2}\left(d_4\right)T_{rot5}\left( {}_{45k}\right)T_{rot5}\left( {}_{45p}\right). \tag{4}$$

$$T_{56} = T_{trans1}\left(\xi_{56k}\right)T_{trans1}\left(\xi_{56p}\right). \tag{5}$$

$$T_{67} = T_{trans1}\left(d_6\right)T_{rot4}\left( {}_{67k}\right)T_{rot4}\left( {}_{67p}\right). \tag{6}$$

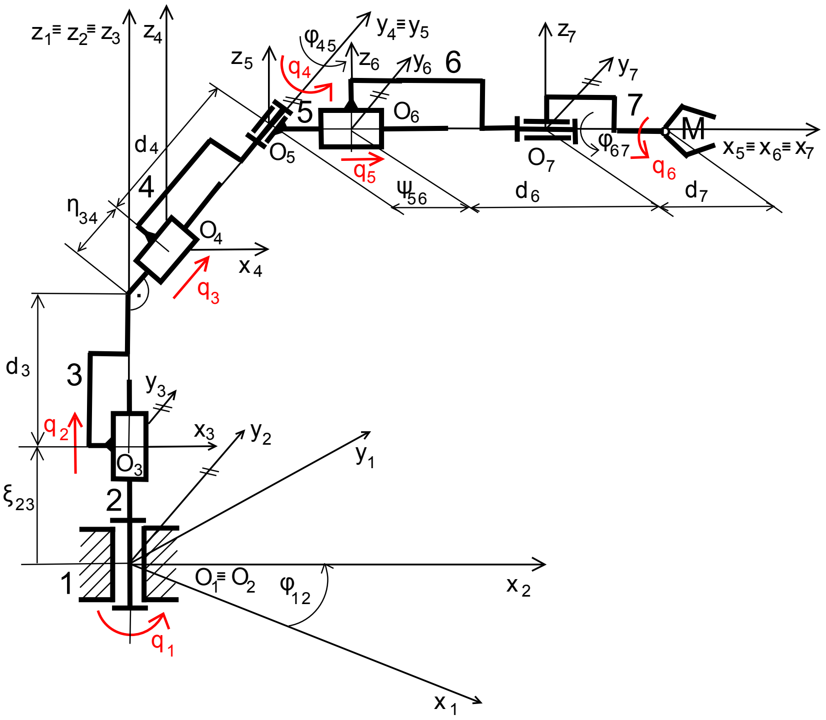


*Fig. 3. Model of the manipulator with 6 DOF*

**Dynamic analysis.** Dynamics of kinematic chains is described by Lagrange equations of the second kind [10; 11]. Generalized coordinates $q_1$, $q_2$, ..., $q_n$ are used to determine the members position. Equations of motion [2] are compiled in the form:

$$\sum_{i=1}^{n} a_{0k}^{i}\ddot{q}_i + \sum_{j=1}^{n}\sum_{i=j}^{n} a_{1k}^{ji}\dot{q}_j\dot{q}_i + a_{2k} = \mathbf{Q}_k,\quad k = 1,\dots,n \tag{7}$$

where:

$$a_{0k}^{i} = \sum_{l=\max(i,k)}^{n} S\left(\mathbf{B}_{l}^{i}\mathbf{I}_{l}\mathbf{B}_{l}^{kT}\right) \tag{8}$$

$$a_{1k}^{ji} = \sigma_{ji}\sum_{l=\max(i,j,k)}^{n} S\left(\mathbf{B}_{l}^{ji}\mathbf{I}_{l}\mathbf{B}_{l}^{kT}\right),\qquad \sigma_{ji} = \begin{cases} 1 & ak\ j = i \\ 2 & ak\ j \neq i \end{cases}\qquad i,j,k = 1,\dots,n \tag{9}$$

$$a_{2k} = gS\left(\Theta_3\left(\sum_{i=k}^{n} m_i\mathbf{B}_{i}^{k}\mathbf{r}_{i}\right)^{T}\right), \tag{10}$$

$$\mathbf{B}_{i} = \mathbf{T}_1\mathbf{T}_2\ldots\mathbf{T}_i = \prod_{k=1}^{i}\mathbf{T}_k \qquad i = 1,\dots,n, \tag{11}$$

$a_{0k}^{i}$ – coefficients of accelerations;

$a_{1k}^{ji}$ – coefficients of velocities;

$a_{2k}$ – coefficients of gravitation.

Equations (7) in matrix notation:

$$\mathbf{A}_0\ddot{\mathbf{q}} + \mathbf{a}_{\dot{\mathbf{q}}} + \mathbf{a}_{\mathbf{g}} = \mathbf{a}_{\mathbf{F}} \tag{12}$$

With modification:

$$\ddot{\mathbf{q}} = \overline{\mathbf{a}}_{\mathbf{F}} - \overline{\mathbf{a}}_{\dot{\mathbf{q}}} - \overline{\mathbf{a}}_{\mathbf{g}} \tag{13}$$

Equations (13) show that the motion of individual members of the kinematic chain have dynamic effect on the other members of the chain. Similarly forces acting on individual members of the kinematic chain have dynamic effect on the other members of the chain. Dynamic coefficients are given by the actual configuration of the kinematic chain.

**Computer analysis of robot models.** Computer modeling can be devided according to the modeling process to mathematical modeling and "multibody" modeling. Mathematical modeling describes the whole model and its behavior using mathematical equations and algorithms.

Mathematical equations defining the motion – equations of motion and equations describing kinematic dependencies between the individual chain members are used to define the properties of the elements of the solved model. However, derivation of these equations is time consuming and laborious and therefore this procedure is only suitable for simple models, for models with a low degree of freedom. Matlab, Maple, Mathematica are kinds of software used here.

Matlab – is an environment that allows numerical computations, modeling and simulation.

Maple – is a comprehensive computational software that allows analytical calculations, numerical calculations, graphical display of results and creation of a document describing the workflow.

Mathematica – is focused on numerical and matrix tasks in various areas of engineering issues.

In multibody modeling we do not directly describe a mathematical model of the whole system. By using multi-body modeling software, we can model the real elements of the system by using a predefined bodies of respective geometry. There are a number of softwares that have different hardware requirements and designer skill requirements. Some of them are Matlab / SimMechanics, Matlab / Simulink, Dynast, MSC Adams.

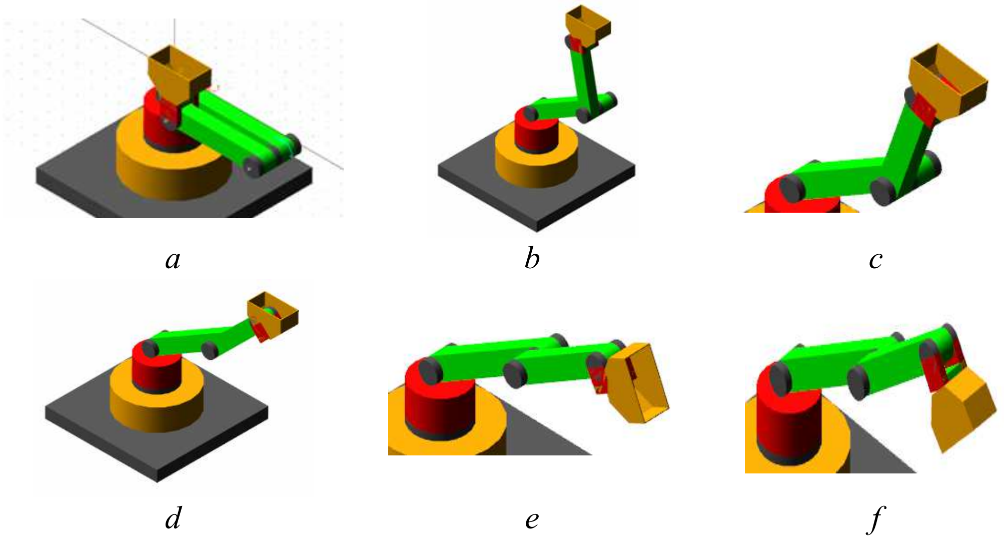
Matlab / SimMechanics – this Matlab extension was developed to address kinematics and rigid body dynamics.

Matlab / Simulink – this Matlab module was developed to create and solve dynamic systems using block diagrams.

Dynast – allows simple mathematical calculations and simulations.

MSC Adams – uses object-oriented programming environment with graphical output. Systems are defined directly by geometry of bodies, kinematic bonds, force effects and motion generators.

**Software realization of the manipulator.** The simulation of manipulator arm movement and the positioning of the basket at its end are shown in Fig. 4 a-f.


*a* *b* *c*

*d* *e* *f*

*Fig. 4. Model of the manipulator*

The position of the basket after processing the results of the simulation in the Postprocessor is shown in Figure 5.

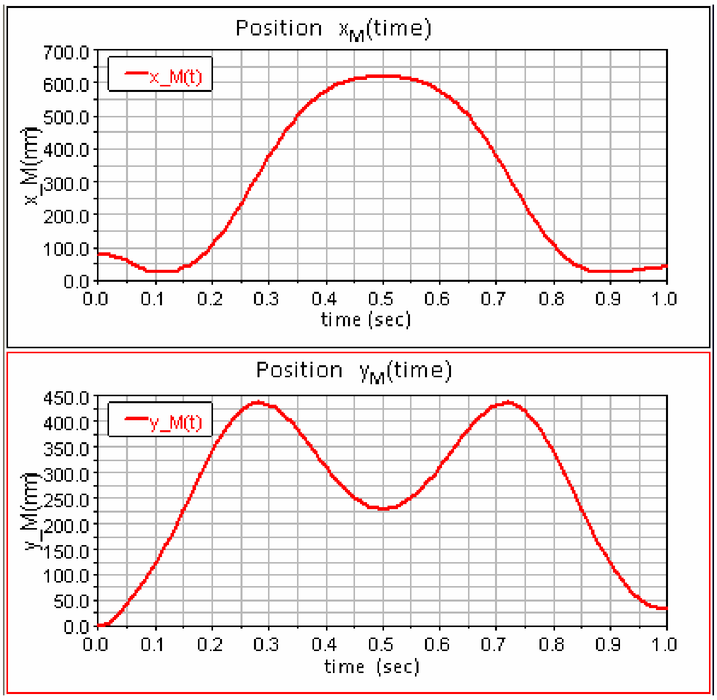


*Fig. 5. Model of the of the manipulator – position $x_M$ and $y_M$*

The trajectory of the end effector of the manipulator is shown in Fig.6.

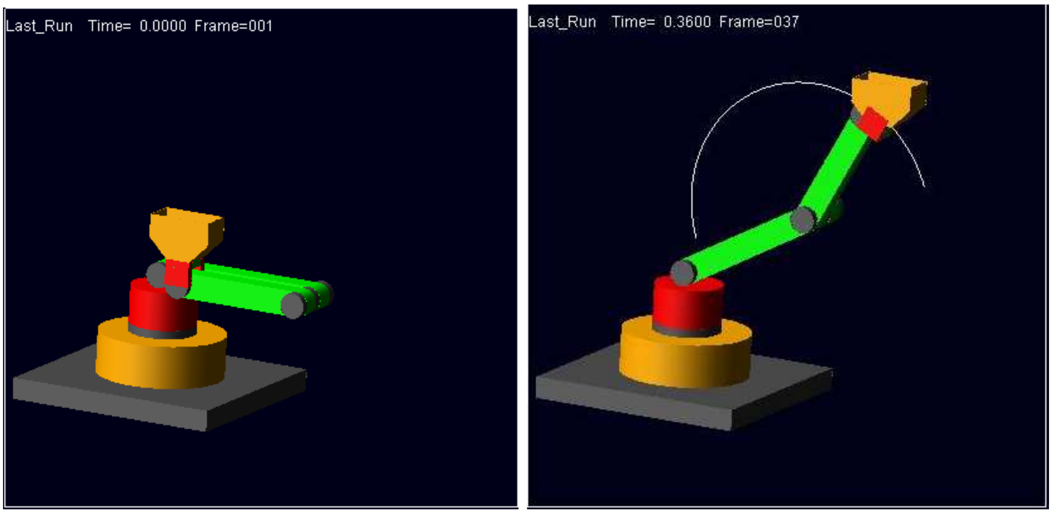


*Fig. 6. Model of the robot and trajectory of the end effector*

The torques in joints of the arms is shown on (Fig. 7) and (Fig. 8).

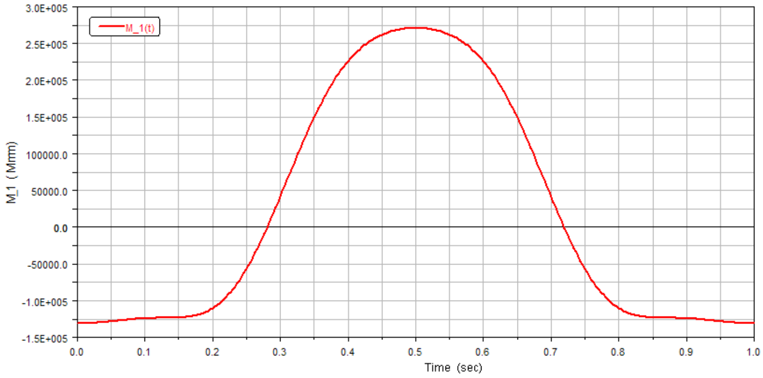


*Fig. 7. The torque M1*

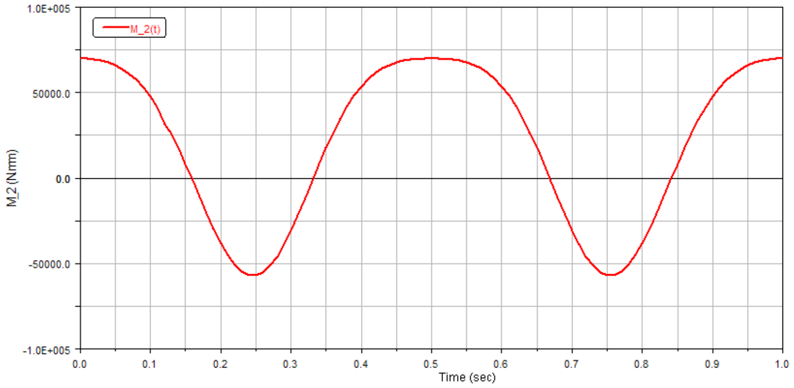


*Fig. 8. The torque M2*

The moments M1 and M2 in rotational joints and the balancing moment for the positioning of the basket of the mobile manipulator after processing in the Postprocessor is shown in Fig. 9.

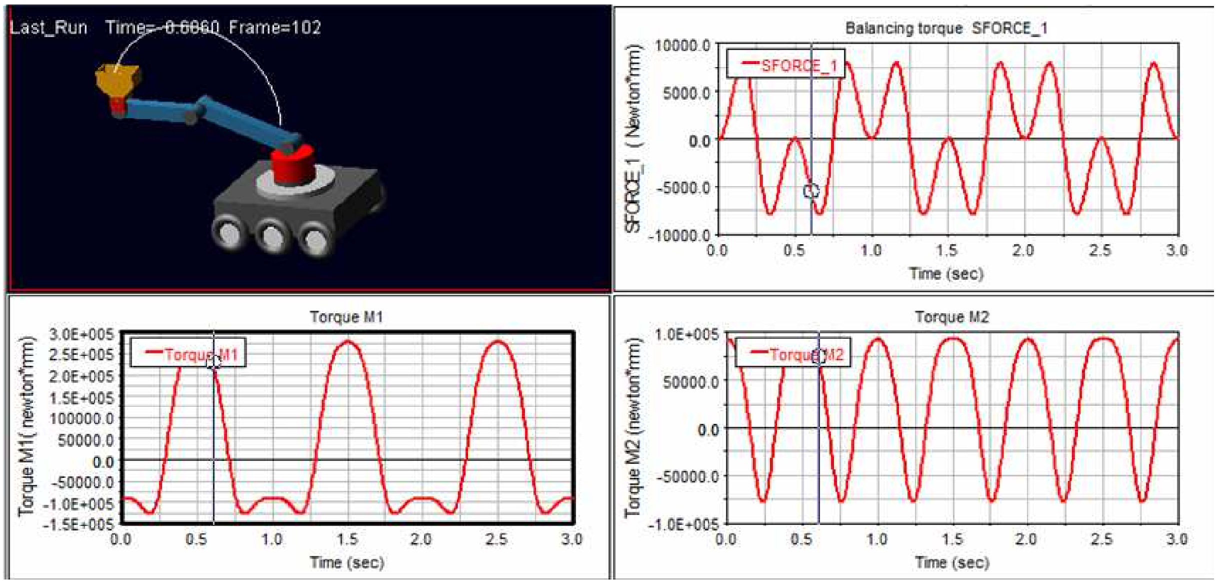


*Fig. 9. Balancing torque SFORCE_1*

Thanks to computer simulation, we can easily modify individual model parameters and track their impact on the results. One of the advantages of computer simulation is the speed and great flexibility in solving various problems [12-17].

**Conclusion.** User friendly environment with graphical output of the simulation allows to define the parts of the model by using a number of basic geometric shapes. Systems are defined directly by geometry of bodies, kinematic bonds, force effects, motion generators. It allows to shorten the test time, reduce the risk of error, obtain input data and the possibility of solving complex mechanical systems with many degrees of freedom.

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## References

1. BRÁT, V. (1981). Maticové metódy v analýze prostorových vázaných systému, Academia, Praha [in English].

2. KOZLOV, V. V., MAKARIČEV, A. V., TIMOFEJEV, E. I., & JUREVIČ, E. I. (1984). *Dynamika riadenia robotov*. Nauka, Moskva [in English].

3. BEJCZY, A. K. (1974). *Robot Arm Dynamics and Control*. NASA ,California [in English].

4. BAKŠI, J. (2017). *Riadenie dvojčlánkového manipulátora s premenlivou zát'ažou*, Diplomová práca, TU Košice [in Slovak].

5. DENAVIT, J. & HARTENBERG, R. S. (1955). *A Kinematic Notation for Lower.* Pair Mechanisms Based on Matrices, Journal of Applied Mechanisms [in English].

6. VITKO, A., JURIŠICA, L., BABINEC, A., DUCHOŇ, F., & KĽÚČIK, M. (2010) *Some Didactic Problems of Teaching Robotics*. Proceedings of the 1st International Conference Robotics in Education. Location: Bratislava, Sept. 16-17, 2010, Bratislava, Slovak University of Technology in Bratislava, 27-30. ISBN 978-80-227-3353-3 [in English].

7. HAUG, E. J. (1984). *Elements and methods of computational dynamics.* Computer Aided Analysis and Optimization of Mechanical System Dynamics, NATO ASI Series, Vol.F9, Springer-Verlag, Berlin, Heidelberg [in English].

8. KOPLIK, J. & LEU, M. C. (1986). Computer Generation of Robot Dynamics Equations and the Related Issues. *Journal of Robotic Systems, 3(3),* 301-319 [in English].

9. STEJSKAL, V., & VALÁŠEK, M. (1996). *Kinematics and dynamics of Machinery.* Marcel Dekker. Inc., New York [in English].

10. SHABANA, A. A. (1998). Dynamics of Multibody Systems (2nd edition), University Press: Cambridge [in English].

11. SHABANA, A. A. (2001). *Computational Dynamics* (2nd edition), John Wiley & Sons, Inc.: New York [in English].

12. VALÁŠEK, M. (1979). *Automatické sestavování obecných pohybových rovnic v symbolické formě.* Výzkumná zpráva, FS-ČVUT [in Slovak].

13. KONIAR, D., HARGAS, L. & SIMONOVA, A. (2014). *Virtual Instrumentation for Visual Inspection in Mechatronic Applications.* 6th Conference on Modelling of Mechanical and Mechatronic Systems. Location: Vysoke Tatry [in English].

14. MIKOVÁ, Ľ., KELEMEN, M., & KELEMENOVÁ, T. (2008). Four wheeled inspection robot with differential controlling of wheels. *Acta Mechanica Slovaca*, *12 (3-B*), 548-558 [in English].

15. DUCHOŇ, F., HUBINSKÝ, P., HANZEL, J., BABINEC, A., & TÖLGYESSY, M. (2012). Intelligent Vehicles as the Robotic Applications. *Procedia Engineering*, *48 (2012*), 105–114. doi.org/10.1016/j.proeng.2012.09.492 [in English].

16. Koniar, D., HARGAŠ, L., & ŠTOFAN, S. (2012). Segmentation of Motion Regions for Biomechanical Systems. *Procedia Engineering*, 48, 304–311. doi.org/10.1016/j.proeng.2012.09.518 [in English].

17. Turygin, Y., & Bozek, P. (2013). Mechatronic systems maintenance and repair management system. *Transfer of innovations*, 26, 3-5 [in English].

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# ПОБУДОВА РІВНЯНЬ І КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ РУХУ БАГАТОЕЛЕМЕНТНИХ СИСТЕМ

***Актуальність теми дослідження.*** *Комп'ютерні моделі формують нову якість процесу пізнання. Із використанням комп'ютерної моделі, властивості досліджуваного об'єкта можуть бути перевірені в різних умовах експлуатації. Експериментуючи з комп'ютерною моделлю, ми дізнаємося про модельований об'єкт. Ми можемо протестувати різні варіанти машин без необхідності створювати та редагувати прототипи.*

***Постановка проблеми.*** *Розвиток комп'ютерних технологій розширив можливості розв'язання математичних моделей і дозволив поступово автоматизувати розрахунок рівнянь математичної моделі. Необхідно вставити відповідні входи математичної моделі, а також моніторити та оцінювати вихідні результати за допомогою пристрою виведення комп'ютера. Мета полягала в тому, щоб описати математичний апарат, необхідний для математичного моделювання, а потім скомпілювати модель для комп'ютерного моделювання.*

***Аналіз останніх досліджень і публікацій.*** *При розробці математичної моделі для комп'ютера закони і теорія, які ми використовуємо, завжди дійсні в більш-менш ідеалізованих умовах і оперують фіктивними поняттями, такими як матеріальна точка, ідеальний газ, нематеріальна пружина тощо. Однак з такими спрощеннями ми описуємо реалістичне явище, коли початкові припущення зустрічаються тільки в певній мірі. Щоб результати не відрізнялися від модельованої реальності, слід припустити, що хороша комп'ютерна модель виникає поступово, шляхом її перевірки і модифікації, що є одним з переваг MSC Adams.*

***Виділення недосліджених раніше частин загальної проблеми.*** *Питання побудови реальної моделі маніпулятора. На підставі наведеного вище моделювання можна побудувати реальну модель.*

***Постановка завдання.*** *Використання MSC Adams для моделювання багатоелементних систем та перевірки придатності для моделювання маніпуляторів та моделей роботів. У різних варіантах побудованої моделі ми можемо контролювати його поведінку в різних режимах роботи.*

***Виклад основного матеріалу.*** *В комп'ютерному моделюванні MSC Adams-View використовується для моделювання механічних систем. Вона має інтерактивне середовище для автоматизованого динамічного аналізу параметризованих механічних систем з довільною структурою жорстких та гнучких тіл з геометричними або силовими з'єднаннями, в яких діють гравітаційні, інерційні, експериментально створений контакт, тертя, аеродинамічні, гідродинамічні або електромеханічні сили і мають встроєні контрольні, гідравлічні, пневматичні або електромеханічні ланцюги.*

***Висновки відповідно до статті.*** *Робота з математичною моделлю на комп'ютері відкриває простір для конкретного синтезу емпіричного та аналітичного методу наукового знання. Робота з комп'ютерною моделлю має характерні риси класичних експериментів. Це представляє якісно новий спосіб вирішення завдань, коли неможливо експериментувати з реальним об'єктом. Результатом є еквівалентність комп'ютерної моделі та досліджуваного об'єкта з вибраними характеристиками та виразами які є необхідні та значимі з точністю, достатньою для конкретної мети.*

***Ключові слова:*** *багатоелементні системи; кінематичні пари; матриці перетворення; моделювання програмного забезпечення.*

*Рис.: 9. Бібл.: 17.*

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