UDC 004.4
DOI: 10.25140/2411-5363-2021-2(24)-33-37

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LINEARIZATION OF A NONLINEAR VEHICLE MODEL

The purpose of this article is to create a mathematical model of a vehicle using dynamic equations of motion and simulation of perturbations acting on a vehicle.

It is assumed that the tire in the car model behaves linearly. Because the vehicle model is nonlinear, the model will need to be linearized in order to find the transfer function between the angle of rotation of the front wheel and the lateral position of the vehicle.

For this purpose, simple dynamic models of the car were created, which reflect its lateral and longitudinal dynamics. These types of models are usually used with a linearized form of mechanical and mathematical equations that are required when designing controllers, active suspension and other driver assistance systems.

Keywords: *linearization; vehicle; nonlinear equations.*

Fig.: 1. Reference: 6.

Introduction. Research in vehicle dynamics has been an ongoing study for decades since the invention of the automobile. Engineers and scientists have sought to fully understand the dynamic behaviour of vehicles that are subjected to a variety of driving conditions, whether it be normal daily driving or extreme emergency manoeuvres. These results will be used to eliminate problems such as ride quality and handling stability to develop innovative designs that will improve vehicle operation.

The nonlinear vehicle model can be linearized by assuming small deviations from steady state. The tyre in the vehicle model is assumed to behave linearly. The linear model is valid at constant longitudinal speed, constant normal tyre load, constant coefficient of road friction and constant longitudinal tyre slip. It is desirable to linearise the vehicle model in real time in order to obtain as accurate model of the vehicle behaviour as possible.

A simple vehicle dynamics models that specifically represent its lateral and longitudinal dynamics have been created for a basic understanding of its dynamic behavior. These types of models are commonly used with a linearized form of mechanical-mathematical equations, which are desirable in the design of controllers, active suspension systems, and other driver assistance systems.

The required complexity of the vehicle model depends on its specific application. For example, simple models containing only in-plane degrees of freedom are sufficient for investigating the handling characteristics of a vehicle during its planar motion. For vehicle stability analysis, two-axle models are used [1-6].

Mathematical model of a two-track vehicle. The two-track vehicle model also includes the vehicle's wheelbase in the calculation.

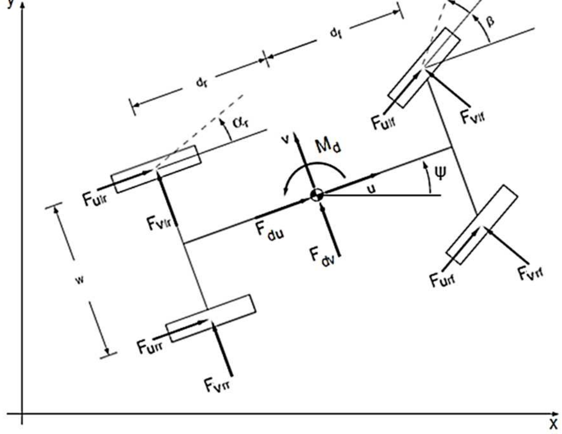

Fig. 1. Two-track vehicle model with indicated positive directions of coordinate systems

The equations of motion are in the form:

$$m.\ddot{u} = F_{ur} + F_{uf}\cos\beta - F_{vf}\sin\beta + F_{Du} \tag{1}$$

$$m.\ddot{v} = F_{vr} + F_{uf}\sin\beta + F_{vf}\cos\beta + F_{Dv} \tag{2}$$

$$J_z\ddot{\psi} = d_f F_{uf}\sin\beta + d_f F_{vf}\cos\beta - d_r F_{vr} + \frac{w}{2}\left(\Delta F_{ur} + \Delta F_{uf}\cos\beta\right) + M_D \tag{3}$$

while:

$$F_{uf} = F_{urf} + F_{ulf} \tag{4}$$

$$F_{ur} = F_{urr} + F_{ulr} \tag{5}$$

$$\Delta F_{uf} = F_{urf} - F_{ulf} \tag{6}$$

$$\Delta F_{ur} = F_{urr} - F_{ulr} \tag{7}$$

For the angles of directional deviations of the wheels defined by global velocities and lateral forces:

$$\alpha_{fr} = \psi + \beta - \tan^{-1}\left(\frac{\dot{y} + \dot{\psi}\left(d_f\cos\psi + \frac{w}{2}\sin\psi\right)}{\dot{x} - \dot{\psi}\left(d_f\sin\psi - \frac{w}{2}\cos\psi\right)}\right) \tag{8}$$

$$\alpha_{fl} = \psi + \beta - \tan^{-1}\left(\frac{\dot{y} + \dot{\psi}\left(d_f\cos\psi - \frac{w}{2}\sin\psi\right)}{\dot{x} - \dot{\psi}\left(d_f\sin\psi + \frac{w}{2}\cos\psi\right)}\right) \tag{9}$$

$$\alpha_{rr} = \psi - \tan^{-1}\left(\frac{\dot{y} + \dot{\psi}\left(-d_r\cos\psi - \frac{w}{2}\sin\psi\right)}{\dot{x} - \dot{\psi}\left(-d_r\sin\psi + \frac{w}{2}\cos\psi\right)}\right) \tag{10}$$

$$\alpha_{rl} = \psi - \tan^{-1}\left(\frac{\dot{y} + \dot{\psi}\left(-d_r\cos\psi + \frac{w}{2}\sin\psi\right)}{\dot{x} - \dot{\psi}\left(-d_r\sin\psi - \frac{w}{2}\cos\psi\right)}\right) \tag{11}$$

$$F_{vij} = C_i\alpha_{ij} \quad i \in (r; l); j \in (r; f) \tag{12}$$

The equations are written for the vehicle coordinate system, but the angles of directional deviation are defined for the global coordinates. Therefore, a rotation matrix is used:

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}\ddot{u}\\ \ddot{v}\end{bmatrix} \tag{13}$$

Once integrated, the vehicle velocity in global coordinates needed to determine the yaw angle is obtained. Further integration yields the global position of the vehicle. When plugged into the equations of motion and simplified for a small wheel angle β, the equations of motion take the form:

$$m\ddot{u} = F_{urr} + F_{ulr} + F_{urf} + F_{ulf} - 2C_f\left(\psi + \beta - \frac{\dot{y} + \dot{\psi}d_f\cos\psi}{\dot{x}}\right)\beta + F_{Du} \tag{14}$$

$$m\ddot{v} = 2C_f\left(\psi + \beta - \frac{\dot{y} + \dot{\psi}d_f\cos\psi}{\dot{x}}\right) + 2C_r\left(\psi - \frac{\dot{y} - \dot{\psi}d_r\cos\psi}{\dot{x}}\right) + \left(F_{urf} + F_{ulf}\right)\beta + F_{dv} \tag{15}$$

$$J_z\ddot{\psi} = d_f\beta\left(F_{urf} + F_{ulf}\right) + 2d_fC_f\left(\psi + \beta - \frac{\dot{y} + \dot{\psi}d_f\cos\psi}{\dot{x}}\right) - 2C_rd_r\left(\psi - \frac{\dot{y} - \dot{\psi}d_r\cos\psi}{\dot{x}}\right) + \frac{w}{2}\left(F_{urr} - F_{ulr} + F_{urf} - F_{ulf}\right) + \frac{w^2\beta\dot{\psi}\sin\psi}{\dot{x}}\left(C_r - C_f\right) + M_D \quad (16)$$

Linearisation of the dynamic equation of motion in the vehicle direction. In equation (14), the nonlinear elements that need to be linearized are found.

$$n_{u1} = \beta\psi \quad (17)$$

$$n_{u2} = \left(\frac{\dot{y} + \dot{\psi}d_f}{\dot{x}}\right)\beta \quad (18)$$

The linearized equation has the form:

$$m\Delta\ddot{x} = 4F_{x0} + F_{du0} + \Delta F_{urr} + \Delta F_{ulr} + \Delta F_{urf} + \Delta F_{ulf} + \Delta F_{Dv} \quad (19)$$

Linearisation of the dynamic equation of motion in the direction perpendicular to the vehicle. The nonlinear terms in equation (15) are:

$$n_{v1} = -\left(\frac{\dot{y} - \dot{\psi}d_r}{\dot{x}}\right) \quad (20)$$

$$n_{v2} = -\left(\frac{\dot{y} + \dot{\psi}d_f}{\dot{x}}\right) \quad (21)$$

$$n_{v3} = \left(F_{urf} + F_{ulf}\right)\beta \quad (22)$$

The linearized equation has the form:

$$m\Delta\ddot{y} = 2(F_{xo} + C_c)\Delta\beta - \frac{4C_c}{\dot{x}_0}\Delta\dot{y} + 4C_c\Delta\psi - \frac{2C_c\left(d_f - d_r\right)\Delta\dot{\psi}}{\dot{x}_0} + \Delta F_{Dv} \quad (23)$$

Linearization of the dynamic equation of motion for vehicle rotation. The nonlinear terms in equation (16) are:

$$n_{\psi 1} = d_f\beta\left(F_{urf} + F_{ulf}\right) \quad (24)$$

$$n_{\psi 2} = -\left(\frac{\dot{y} + \dot{\psi}d_f}{\dot{x}}\right) \quad (25)$$

$$n_{\psi 3} = \left(\frac{\dot{y} - \dot{\psi}d_r}{\dot{x}}\right) \quad (26)$$

The linearized equation has the form:

$$J_z\Delta\ddot{\psi} = 2d_f(F_{xo} + C_c)\Delta\beta + \frac{2C_c\left(d_f - dr\right)}{\dot{x}_0}\Delta\dot{y} + 2C_c\left(d_f - d_r\right)\Delta\psi - \frac{2C_c\left({d_f}^2 + {d_r}^2\right)}{\dot{x}_0}\Delta\dot{\psi} + + \frac{w}{2}\left(\Delta F_{xrr} - \Delta F_{xlr} + \Delta F_{xrf} - \Delta F_{xlf}\right) + \Delta M_D \quad (27)$$

The resulting linearized equations. The linearized vehicle equations of motion (19), (23) and (27) are now summarized. The Δ symbols are omitted for clarity.

$$m\ddot{x} = 4F_{x0} + F_{du0} + F_{urr} + F_{ulr} + F_{urf} + F_{ulf} + F_{Dv} \quad (28)$$

$$m\ddot{y} = 2(F_{xo} + C_c)\beta - \frac{4C_c}{\dot{x}_0}\dot{y} + 4C_c\psi - \left(m\dot{u}_0 + \frac{2C_c(d_f - d_r)}{\dot{x}_0}\right)\dot{\psi} + F_{Dv} \quad (29)$$

$$J_z\ddot{\psi} = 2d_f(F_{xo} + C_c)\beta + \frac{2C_c(d_f - dr)}{\dot{x}_0}\dot{y} + 2C_c(d_f - d_r)\psi - \frac{2C_c({d_f}^2 + {d_r}^2)}{\dot{x}_0}\dot{\psi} + \\ + \frac{w}{2}(F_{urr} - F_{ulr} + F_{urf} - F_{ulf}) + M_D. \quad (30)$$

Conclusion. Simulation models are widely used in all industries. Their main goal is to understand what behaviour can be expected from a real system. The aim of this paper was the linearization of a two-track vehicle. In most cases, the behavior of dynamic systems is determined by nonlinear differential equations. Since in this case it is not possible to use the Laplace transform to express the image transfer, it is necessary to linearize the dynamical system in question. The essence of linearization of nonlinear dynamical systems is the assumption that the input and output quantities are varied so that their deviations from the steady state are sufficiently small, which is valid for this case. This linearized model will be further applied to the lateral control of a vehicle.

ACKNOWLEDGEMENTS

This research was funded by Slovak Grant VEGA 1/0389/18 “Research on kinematically redundant mechanisms”.

References

1. Grau, C. A, (2002). A parametric study of the lateral dynamics of a nonlinear fourwheel road-vehicle model. *Thesis*. Cincinati: University of Cincinati.

2. Lyshevski, S. E., Nazarov, A. (2000). Lateral maneuvering of ground vehicles. *Modeling and control, Proceedings of the 2000 American Control Conference. Chicago*, American Automatic Control Council.

3. Rajamani, R. (2012). Vehicle dynamics and control. *New York: Springer*, pp. 496, ISBN: 978-1-4614-1433-9.

4. Rossetter, E. J., Gerdes, J. Ch. A study of lateral vehicle control under a ‘virtual’ force framework. *Design Division*, Department of Mechanical Engineering, Stanford University, Stanford, California.

5. Sienel, W. (1997). Estimation of the tire cornering stifness and its application to active car steering. *Proceedings of the 36th Conference on Decision & Control*. San Diego.

6. Yih, P., Jihan R., Gerdes. J.CH. (2003). Modification of vehicle handling characteristics via steer-by-wire. *American Control Conference*. Proceedings of the 2003. Vol. 3. IEEE.

UDC 004

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ЛІНЕАРИЗАЦІЯ НЕЛІНЄЙНОЇ МОДЕЛІ ТРАНСПОРТНОГО ЗАСОБУ

Дослідження динаміки автомобілів тривають із моменту винаходу автомобіля. Інженери та вчені прагнули детально зрозуміти динамічну поведінку транспортних засобів у різноманітних умовах керування, або це звичайне щоденне керування, або екстремальні маневри.

Метою цієї статті є створення математичної моделі транспортного засобу з використанням динамічних рівнянь руху та моделювання збурень, що діють на транспортний засіб із навколишнього середовища. Оскільки модель транспортного засобу нелінійна, для того щоб знайти передавальну функцію між кутом повороту переднього колеса та бічним положенням автомобіля, необхідно буде лінеаризувати модель.

Ця лінеаризована модель транспортного засобу буде додатково застосована при керуванні автотранспортним засобом із заносом.

Для базового розуміння його динамічної поведінки були створені прості динамічні моделі автомобіля, які відображають його бічну та поздовжню динаміку. Такі типи моделей зазвичай використовуються з лінеаризованою формою механіко-математичних рівнянь, які потрібні для проєктування контролерів, систем активної підвіски та інших систем допомоги водієві.

Нелінійні рівняння руху, які описують рух транспортного засобу в напрямку транспортного засобу, у напрямку, перпендикулярному транспортному засобу, та рівняння руху під час повороту автомобіля будуть лінеаризовані.

Метою цієї роботи була лінеаризація двоколійного транспортного засобу. Поведінка динамічних систем визначається нелінійними диференціальними рівняннями. Оскільки в цьому випадку неможливо використати перетворення Лапласа, необхідно лінеаризувати динамічну систему, про яку йдеться. Суть лінеаризації нелінійних динамічних систем полягає у припущенні, що вхідні та вихідні величини змінюються так, що їх відхилення від стаціонарного стану достатньо малі, що справедливо для даного випадку. Ця лінеаризована модель буде додатково застосована при керуванні транспортним засобом із заносом.

Ключові слова: *лінеаризація; транспортний засіб; нелінійні рівняння.*

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Miková, Ľ. (2021). Linearization of a nonlinear vehicle model. *Technical sciences and technologies,* (2(24)), 33-37.