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THE MAIN PERIODS OF EVOLUTION OF THE CONJUGATE ACTION LAW: IN PARALLEL-AXES GEARING

Three fundamental laws of gearing are distinguished in the modern theory of gearing. They are listed immediately below [3]:

1. *The Law of Contact* between interacting tooth flanks: The first fundamental law of gearing.
2. *Epy Conjugate Action Law* between interacting tooth flanks: The second fundamental law of gearing.
3. *The Law of Equal Base Pitches*: The third fundamental law of gearing.

The set of fundamental laws of gearing is also relevant to the field of gear cutting tool design [4], especially to design of generating gear cutting tools: hobs, shapers, shavers, and so forth.

All three fundamental laws of gearing are of critical importance for the theory of gearing, as following to them makes possible designing geometrically-accurate gears for quiet gears that feature the highest possible power-density.

Below, in this text, the readers' attention is focused mainly on the main periods of evolution of the conjugate action law in parallel-axes gearing.

An *equivalent pulley-and-belt transmission* (Fig. 1) can be used to transmit smoothly a uniform rotation from a driving shaft to a driven shaft. If we are about to replace the *equivalent pulley-and-belt transmission* with a gear pair, the gear tooth profiles have to be designed so, as to ensure a straight motion of contact point along the line of action. Under such a scenario, the *gear pair* and the *pulley-and-belt transmission* become equivalent to one another (except of the direction of rotation – the direction of the rotation turns to the opposite).

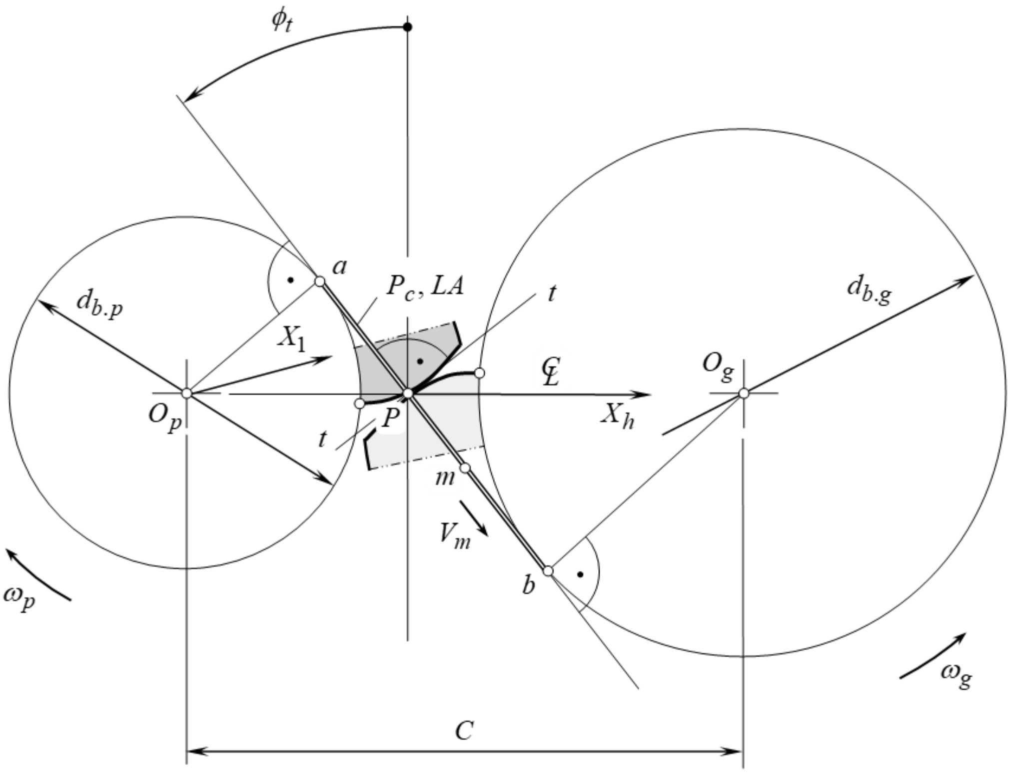

Fig. 1. – Second fundamental law of gearing (conjugate action law): Generation of natural form of a gear tooth profile in geometrically-accurate parallel-axes gearing.

Three main periods of evolution of the conjugate action law in parallel-axes gearing are recognized here. They are as follows:

1. *Pre-Eulerian period* of evolution of the conjugate action law
2. The period of evolution when the *Conjugate Action Law* was discovered, and has been understood in details
3. *Post-Eulerian period* of evolution of the conjugate action law

In the *pre-Eulerian period*, the names of *Girard Desargus* (1591-1661), *Philippe de La Hire* (1640-1718), and *Charles Camus* (1699-1768) deserve to be mentioned. However, only research undertaken by *Camus* is of importance in this regard. Despite of *Camus* was close to discover the conjugate action law, he failed to make the final, but very important step: *Camus* didn't differentiate between the line of action in a gear pair, and between the path of contact.

Leonhard Euler (1707-1783) was the first to realize the importance of the conjugate action law. The research by *Euler* is published in his two famous papers [1], [2]. A few decades later, *Felix Savary* (1797-1841) came up with the equivalent results of the research. These results were represented in a form that is close to what we know now about the conjugate action law in parallel-axes gearing.

Taking into account the contribution by *Camus*, *Euler*, and by *Savary*, the conjugate action law is often referred to as *Camus-Euler-Savary fundamental law of parallel-axes gearing* (or just *CES fundamental law of parallel-axes gearing*).

Later on, in his 1842 book, *Theodore Olivier* (1793-1853) determined tooth profiles of a gear and of a mating pinion using for this purpose the envelope condition. This is a huge mistake [5] that later was committed one more time by *Ch. Gochman* (1851-1916), who used analytical methods of analysis.

A famous mechanician and scientist *Franz Reuleaux* (1829-1905) is credited with the development of the so-called *method of common perpendiculars*. The proposed method is correct when no rolling motion is observed, and is incorrect when rolling motion is present. Therefore, the proposed by *Reuleaux* method is not applicable to gears and gearing.

Professor *V.A. Shishkov* contributed a lot to the *method of common perpendiculars*. Unfortunately, he also recommended to use this method in cases when rolling motion occurs. Therefore, the results obtained by *Shishkov* are incorrect, and are not applicable to gears and gearing.

No gears with non-involute tooth profile (cycloid gears, straight-sided splines, gears for *Novikov*/conformal/high-conformal gear pairs, and so forth) can be properly designed, and manufactured if the conjugate action law is ignored. It is wrong practice to replace the conjugate action law by the condition of contact of two smooth regular surfaces (that is, by the *Shishkov equation of contact*, $\mathbf{n}_g \cdot \mathbf{V}_\Sigma = 0$).

References

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5. Радзевич С.П., "Новый результат в классической теории огибающих кривых и поверхностей", *Вестник научно-технического развития*, №8(156), 2020, с. 12-21, www.vntr.ru.