

Lecture Notes in Mechanical Engineering

Vitalii Ivanov · Justyna Trojanowska ·  
Ivan Pavlenko · Erwin Rauch ·  
Ján Pitel' *Editors*

# Advances in Design, Simulation and Manufacturing VI


Proceedings of the 6th International  
Conference on Design, Simulation,  
Manufacturing: The Innovation  
Exchange, DSMIE-2023,  
June 6–9, 2023, High Tatras, Slovak  
Republic - Volume 1: Manufacturing  
Engineering

 Springer

# Lecture Notes in Mechanical Engineering

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June 6–9, 2023, High Tatras, Slovak Republic -  
Volume 1: Manufacturing Engineering

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ISSN 2195-4356

ISSN 2195-4364 (electronic)

Lecture Notes in Mechanical Engineering

ISBN 978-3-031-32766-7

ISBN 978-3-031-32767-4 (eBook)

<https://doi.org/10.1007/978-3-031-32767-4>

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# Surface Relief Formation in Peripheral End Milling

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**Abstract.** Theoretical studies of peripheral milling and the results of simulating the construction of a 3D relief of the machined surface have been presented. A digital model of the surface relief formation during peripheral milling with an end mill has been developed. The related simulating algorithm has been created. The milling process model is compiled taking into account the dynamic phenomena occurring in a closed technological machining system (TMS) and machining along the trace, represented by a function of a delay argument. The dynamic system is represented by a single-mass model with two degrees of freedom. The created application program for simulating peripheral milling with an end mill makes it possible to predict the power parameters of the cutting process and the geometric and machining parameters, depending on the geometric parameters of the cutter and workpiece, cutting mode, and dynamic characteristics of TMS. A 3D relief of the machined surface has been constructed. Its geometric component consists of the shape of micro-roughness, formed from the geometric engagement of the cutting edge of the helical mill with the workpiece and the elastic movement of the TMS. Simultaneously, it corresponds to the interaction of the next cutter tooth along the helix. The created application program makes it possible to predict the microrelief of the machined surface depending on the main TMS parameters.

**Keywords:** End Milling · Simulation Machining · Machined Surface Relief · Process Innovation

## 1 Introduction

Milling operations are widely used for the machining of parts in various industries. The purpose of such processes is not only to obtain the required geometric shape accuracy but also to the machined surface’s quality, which is estimated by its roughness. A complete assessment of the quality of the machined surface can be made based on the analysis of its 3D relief [1]. Surface roughness is formed due to the engagement of the cutting edges of the mill with the workpiece. Such engagement is carried out in a closed elastic technological machining system (TMS) and is determined by the cutting mode and the dynamic parameters of TMS. Understanding the formation mechanism of surface roughness during the milling operation is crucial for predicting the influence of TMS dynamic parameters and the correct choice of cutting mode [2].

The relief of the machined surface consists of two different components [3]. The first is the ideal or geometric component, which results from the geometric interaction between the tool's cutting edges and the workpiece. The ideal component of roughness in peripheral milling can be calculated from the cutter's geometrical parameters, the machined surface's curvature, and the cutting mode. The second component is a natural or random component that occurs as a result of tool wear, vibration, and dynamics of the cutting process and the effects of the material being processed. Unlike the ideal component, the natural finish is generally difficult to predict. It is often the dominant finishing component when machining steel and other hard materials with carbide tools or inhomogeneous materials such as cast iron or powdered metals [3].

The formation of the surface relief is carried out in the process of cutting. Fluctuations in the relative position of the tool and the workpiece accompany it. Therefore, the process dynamics significantly impact the formation of the surface relief [4]. The formation of surface relief during 3D circumferential end milling suggests that the dynamic deviation of the tool and workpiece in the direction of the longitudinal axis of the cutter can also be attributed to the ideal component. Therefore, the development of research in the direction of taking into account the influence of dynamic phenomena on the formation of roughness is an essential task for improving the adequacy of models and controlling the end milling process.

## 2 Literature Review

The modulating component in the surface relief is ideal. The solution to the problem of predicting the roughness of a machined surface [5] can be based on a mathematical model of the peripheral milling process. The process model should consider the TMS's closed nature, the presence of at least two elastic systems in the relief formation plane, and processing along the trail.

To simulate the milling process, taking into account machining along the trace, it is necessary to determine the geometry of the machined surface, both taking into account the formation of micro-roughness during the engagement of the cutting edge with the workpiece and the vibration of the TMS. These two processes co-occur and form a layer of allowance for the next cutter tooth. In works [6, 7] devoted to vibrations during milling, one can find an image of such a surface presented graphically without considering the interaction of these two processes. It should be noted that the relief of the machined surface is schematically depicted on various machining schemes in the form of a wavy line but is not represented as a result of computer simulation of the milling process. Therefore, the adequacy of this representation is also questionable.

Peripheral milling forms a 3D surface, the relief of which is determined by considering the parameters of the helical surface of the cutting edge of the mill. It is proposed to evaluate the surface with new parameters, such as the maximum error in determining the surface, the total profile height in the feed direction, and the total profile height in the axial direction [8]. However, the generated models represent the machined surface for the range of stable cutting parameters.

To study the roughness of the machined surface, statistical methods are widely used, for example, using the Taguchi optimization method [9]. The model for the face milling

process is based on cutting process parameters such as feed, spindle speed, depth of cut, and radial immersion. Surface roughness data were collected for nine experiments. The experiments made it possible to establish the signal-to-noise ratio and calculate the variance to determine the optimal level and percentage of the contribution of each parameter. However, the resulting models are not universal and depend entirely on the conditions of a particular machining process.

Despite such limitations, sometimes the method of studying the process of roughness formation, based on a statistical analysis of the experimental results, is effective. To ensure maximum prediction accuracy, it is proposed to use three types of intelligent networks. These are neural networks with a radial basis function, adaptive neuro-fuzzy inference systems, and genetically developed fuzzy inference systems [10]. Despite such an arsenal of research, the results obtained are not much different from the known models compiled based on statistical data processing.

One can observe this direction's development using machine vision [11]. A system for automated, non-contact, and flexible prediction of the surface roughness of end-milled parts using a machine vision system integrated with an adaptive neuro-fuzzy inference system (ANFIS) is presented. The images of the milled surface obtained by the machine vision system were processed using a 2D Fourier transform to get image texture characteristics. Cutting speed, feed, and depth of cut were taken as input parameters, and surface roughness as output parameters.

However, such approaches do not explore the mechanisms of relief formation. Therefore, they have a common drawback that does not allow one to distinguish components of different natures in the overall relief of the treated surface. Therefore, they cannot differentiate process control methods to reduce these components. In addition, it should be noted that the formation of the surface in each section of the cutter is carried out by one tooth, i.e., its peak, which is also insufficiently represented in the process modeling.

### **3 Research Methodology**

#### **3.1 The Aim and Objectives of the Study**

To develop a 3D model of the relief formation of the surface of a part machined by peripheral milling with an end mill, taking into account dynamic phenomena in a closed technological machining system.

To achieve this objective, it is necessary to solve the following tasks:

1. Compile a digital model of the surface relief formation during peripheral milling with an end mill, considering the dynamic phenomena occurring in a closed technological machining system.
2. Create an application program for modeling the process of peripheral milling with an end mill with the identification of force and geometric factors during cutting.
3. Provide the formation of a 3D relief model of the machined surface, considering the end mill's helical cutting edge.



### 3.2 Digital Model of Surface Relief Formation During Peripheral Milling

Determining the surface topography during peripheral milling with an end mill can be performed by digitally modeling the entire cutting process according to the algorithm. The enlarged diagram is shown in Fig. 1. The algorithm repeats the procedures with a discrete step  $\delta\varphi$  (rad) of the rotation angle for the mill and the corresponding displacement  $\delta X$  (mm) of the cutter center in the feed direction. These movements are synchronized in time:

$$\delta\phi = 33.3V\delta t/D, \quad \delta X = f\delta t/60 \quad (1)$$

where  $V$  is the cutting speed, (m/min),  $D$  is the mill diameter, (mm),  $f$  is the feed, (mm/min),  $\delta t$  is the time step, (s).

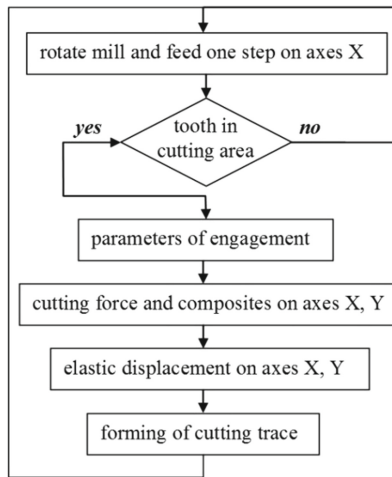


Fig. 1. Algorithm general of simulation.

In the simulation, a particular procedure is used to determine the starting point of cutting. When the tooth tip enters the zone of interaction with the workpiece, the method for determining the parameters necessary for calculating the cutting force is performed. Considering the repeatability of the algorithm for each step, the calculation of the cut layer thickness  $a$  is performed by recursive relations as a function of the angle  $\varphi$ :

$$a_i = \sqrt{(y_{i-1} - Oy_i + R_m \cos\phi_i)^2 + (x_{i-1} - Ox_i + R_m \sin\phi_i)^2}, \quad (2)$$

where  $y_{i-1}, x_{i-1}$  are the coordinates of the workpiece surface point formed on the previous pass,  $Oy_i, Ox_i$  are the coordinates of the cutter center on the current pass,  $R_m$  is the mill radius.

To calculate the components of the cutting force, it is convenient to use the dependencies based on the linearization of the cutting force. It is possible to express the cutting force on any cutting edge as a function of chip area and specific force [4]:

$$F_i = K_s b a_i, \quad (3)$$

where  $K_s$  is the specific cutting force  $b$  is the cutting width.

Taking into account that the tangential component acts along the normal to the front surface of the cutting wedge of the cutter tooth, it is possible to determine the components of the cutting force along the coordinate axes [12]:

$$F_x = FCos(\phi - \gamma - \beta), \quad F_y = FSin(\phi - \gamma - \beta), \tag{4}$$

where  $\gamma$  – rake angle  $\sim 10^\circ$ , and angle  $\beta = \arctan(F_n/F_t) = \arctan(k_{nc}/k_{tc}) = \arctan(0.5) = 26.5^\circ$ .

These components of the cutting force are the perturbing effects of the dynamic system, which can be represented by a single-mass system with two degrees of freedom in a plane perpendicular to the cutter axis:

$$\begin{cases} \frac{s^2 O_x}{\omega_x^2} + 2\xi \frac{s O_x}{\omega_x} + O_x = \frac{1}{k_x} F_x \\ \frac{s^2 O_y}{\omega_y^2} + 2\xi \frac{s O_y}{\omega_y} + O_y = \frac{1}{k_y} F_y \end{cases} \tag{5}$$

where  $\omega_x, \omega_y$  – the frequencies of natural vibrations along axes X and Y;  $O_x, O_y$  – elastic displacements of mill center along the corresponding coordinate axes;  $k_x, k_y$  – the stiffness,  $\xi$  is damping coefficient;  $s$  – the Laplace operator.

After determining the actual position of the mill, the tooth tip path forms the machined surface of the workpiece. This surface will be the initial one when simulating the next cutter tooth pass. This is how the lagging argument function is implemented, which plays an essential role in simulating the stability of a TMS.

Next, the interaction of the helical cutting edge of the cutter tooth is determined according to the angle of inclination of the helical groove of the mill. To simulate this interaction, the mill is represented by its sections as elementary cylindrical mills. This is how the numerical method of integrating the double integral is implemented, presented in [13]. The top of the tooth of each elementary cutter in each section along the milling width forms the relief of the machined surface. Therefore, changes in the position of the center of the cutter, determined at each moment according to the model (5), will affect the machined surface shape in the Z-direction (Fig. 2).

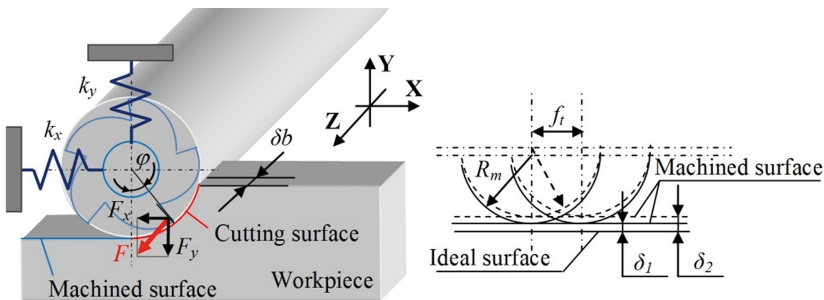


Fig. 2. Relief forming.

On Fig. 2 shows the position of the tool surface of the cutter when moving by the amount of feed per tooth in one cross-section. The relief line of the machined surface

differs from the ideal one by the value  $\delta_1$  of elastic deformation, which is determined by the amplitude of the transient process at this point. The tooth position of the cutting edge forms the surface in cross-section at a distance  $\delta b$ . It differs from the ideal surface by the amount  $\delta_2$  of elastic deformation. Thus, the response of the elastic system to a force perturbation will participate in the relief formation of the machined surface in the Z-direction. The discretization along the Z axis is related to the step  $\delta t$  of the simulation and the cutting mode:

$$\delta b = \omega \delta t R_m / \tan \beta \quad (6)$$

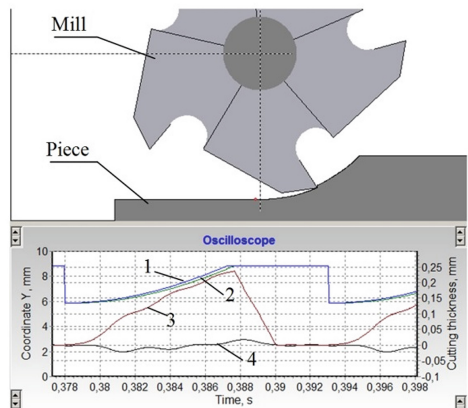
where  $\omega$  – the angular velocity of the mill;  $R_m$  – the radius of the cutter;  $\beta$  – the angle of inclination of the helical groove.

This approach makes it possible to determine the 3D relief of the machined surface.

### 3.3 Simulation

To simulate the process of peripheral milling, an application program was created on object-oriented language that provides the possibility of forming a surface relief, including in 3D measurement. The height of the roughness of the machined surface in each section along the milling width is calculated numerically using an algorithm that will determine the surface of the workpiece, taking into account the geometric and dynamic parameters of the process. The simulation step is 0.00002 s, which makes it possible to observe the fast dynamic processes of an actual system and the shape of the micro-roughness of the machined surface.

Figure 3 shows the process simulation results with input data corresponding to actual 2.5D peripheral milling parameters: X-axis stiffness 6000 N/mm, main natural frequency 340 Hz, Y-axis stiffness 6500 N/mm, and main natural frequency 380 Hz.



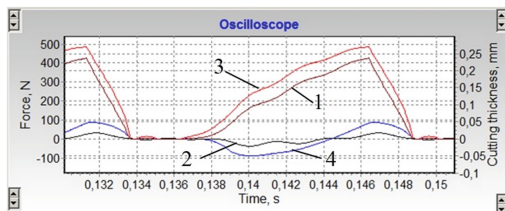
**Fig. 3.** Simulation results: 1 – machined surface, 2 – machining surface, 3 – cutting thickness, 4 – elastic displacement on Y.

The frequencies of natural oscillations and the damping coefficient of oscillations were determined from the experimental data according to the procedure, which utilizes

signal from impact hammer model 086C03 PCB Piezotronics. The tool chosen was a  $\varnothing 20$  mm end mill with five teeth. Cutting mode: cutting speed 50 m/min, feed 1500 mm/min, cutting immersion 3 mm, cutting width 5 mm.

On the oscillograms of the process, line 1 shows the cutting surface machined in the previous pass of the cutter tooth, and line 2 shows the cutting surface in the current pass for the first section along the mill axis. According to the accepted concept of simulation, considering the lagging argument, the thickness of the cut layer is calculated by formula (2) regarding the difference between the two cutting surfaces. The thickness of the cut layer of the allowance is indicated by line 3. It can be seen that the thickness of the cut layer is affected by fluctuations in the elastic TMS (line 4). Its calculation considers the helical cutting edge of the mill tooth and the milling width, cutting immersion, by numerical integration of the double integral [13]. Therefore, the cutting process is performed by other cutting edges of the helical cutting edge of the mill after the cutting process in the first section has stopped. Elastic fluctuations of TMS through changes in the thickness of the cut layer form a new cutting surface, which will be perceived as already machined. Thus, machining will be carried out along the trace in the simulation. It should be noted that the coordinates of the cutting surfaces are measured on the correct scale of the virtual oscilloscope, and the thickness of the cut layer and elastic displacement are measured on the left scale.

The program allows observing the cutting force components considering the influence of TOC elastic oscillations. Through a change in the thickness of the cut layer, they cause the corresponding force reaction of the cutting process (Fig. 4). Thus, during modeling, one of the main properties of TOS is realized in a closed loop.



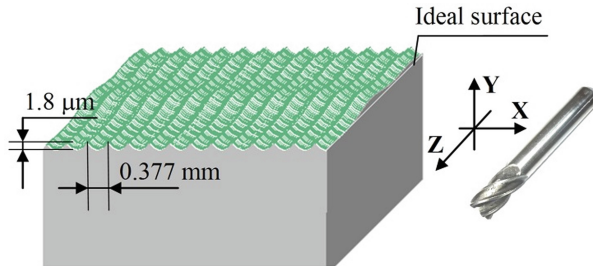
**Fig. 4.** Cutting force in the process: 1 – cutting thickness, 2 – elastic displacement, 3 – composite X cutting force, 4 – composite Y cutting force.

### 3.4 Formation of a 3D Relief Model of the Machined Surface

The machined surface is formed by one tooth of the helical mill edge of the cutter in one section along the width, which corresponds to the Z axis during simulation. Since, with a stable cutting process, steady oscillations arise in the elastic TMS, which means that the form of elastic displacement and the magnitude of the amplitude are repeated for each next tooth.

Therefore, the 3D relief of the machined surface will consist of the shape of micro-roughness. It is formed due to the geometric interaction of the cutting edge of the helical mill with the workpiece and the elastic movement of the TMS at the time corresponding

to the interaction of the next cutter tooth along the helix. Thus, to simulate a 3D relief, it is necessary to add the TMS elastic displacement to the geometric shape of the contour (see line 2 in Fig. 4) according to formula (6).

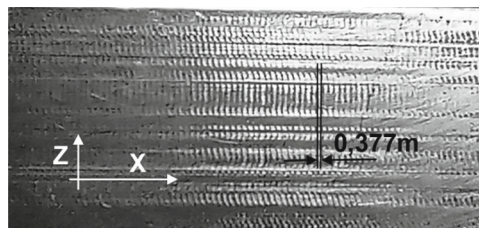


**Fig. 5.** 3D relief of machined surface.

On the side surface of the part, the line shows the location of the ideal surface (Fig. 5). In the direction of the X-axis, the machined surface differs from the ideal one by the amount of micro-roughness. In the Z-direction, each contour differs by the amount of elastic deformation at the time of formation.

The created application program will allow you to simulate the process of peripheral milling and observe all its main characteristics synchronously with the animation of the graphic image of the process. It is possible to evaluate the influence of both the geometrical parameters of the process, the cutting mode, and the dynamic characteristics of TMS.

To confirm the results obtained, an experiment of end milling of the surface was carried out with parameters entirely consistent with the simulation. Figure 6 shows a fragment of the treated surface on an enlarged scale.



**Fig. 6.** Plan machined surface.

Microroughness of the relief is observed, formed due to the geometric interaction of the tool edge and the workpiece, the step of which corresponds to the cutting mode with a milling cutter  $\varnothing 20$  mm  $z = 5$ . The surface relief in the direction of the Z axis is subject to the influence of technological system chatter, which leads to a change in the height of the main surface roughness component according to the simulation results in Fig. 5.

## 4 Results and Discussion

The simulating algorithm has been developed to implement a digital model of the peripheral milling process. It allows for determining all the main characteristics in the time range (Fig. 1). The proposed model's peculiarity is that it considers three main properties of the cutting process. Firstly, the cutting process is carried out in a closed loop TMS, the structure of which is covered by feedback in the direction of two coordinates in the form of elastic displacements under the action of the cutting force. At the same time, these displacements lead to a change in the geometric arrangement of the mill and the workpiece. This fact changes the thickness of the cut layer and, accordingly, the cutting force. This is the closeness of the cutting process in elastic TOS. Secondly, the cutting process is simulated, considering the machining along the trace when deforming the cutting surface trajectory and determining the cut layer's thickness (2). Thirdly, the linearization of the cutting force (3) and its decomposition into components (4) makes it possible to model the dynamics of the process by a system of 4th-order linear differential Eqs. (5). The model is solved numerically using the standard 4th-order Runge-Kutta integration procedure.

The developed approach to modeling, when the definition of the main characteristics and its visualization is performed synchronously on the same interface, allows you to associate the process parameters with a specific moment of interaction between the cutting edge of the cutter and the workpiece (Fig. 3).

The simultaneous representation of the process of removing the allowance in the form of the engagement of the mill tooth with the cutting surface and the already machined surface made it possible to confirm the hypothesis of relief formation. This changes some existing models [6, 7] when the machined surface in the section is presented with traces of vibrations caused by dynamic phenomena, which is untrue. Such representations were made due to the lack of the machining simulation in time with the visualization of the allowance removal. Traces of vibrations will necessarily be present on the cutting surface, which provokes machining along the trace, which is simulated by a delay argument. This is the main novelty of the presented research. The proposed new approach to determining the relief of the machined surface made it possible to construct a 3D digital copy of the machined surface (Fig. 5).

The created simulation program allows for predicting the relief shape of the machined surface, depending on the geometric parameters of the cutting process, the dynamic parameters of TMS, and the cutting mode for the process of peripheral milling of a flat surface. However, the related principles can be extended to the machining model for both convex and concave surfaces. Also, expanding the possibilities for predicting the relief during contour milling of 2.5D surfaces on a CNC machine using a control program presented in G-codes will significantly increase the practical value of the development.

The adequate results confirmation of the dynamics influences studies of the machining in the end milling on the relief of the surface machined is supposed to be obtained using 3D scanning on a microscope, for example, the Keyence VHX-7000 series. Such studies will improve the developed simulation program and increase its efficiency in predicting the results of end milling, especially for non-rigid systems.

## 5 Conclusions

The digital model of the formation of a surface relief during peripheral milling with an end mill has been developed. A simulating algorithm has been created considering the dynamic phenomena occurring in a closed technological machining system, making it possible to determine a 3D model of the machined surface.

The application program for the simulation of the peripheral milling process with an end mill has been created. It makes it possible to predict the power parameters of the cutting process and the geometric parameters of the machining and machined surface, depending on the geometric parameters of the mill and workpiece, cutting mode, and dynamic characteristics of TMS.

It has been proved that the contour of the machined surface relief in the cross-section consists of the micro-roughness of the geometric component. It is formed during the interaction of the helical cutting edge tooth of the mill, the middle line of which deviates from the ideal one by the amplitude of the TMS oscillations at the corresponding moment of cutting.

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